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NON-EUCLIDEAN GEOMETRY: HISTORICAL AND EXPOSITORY.

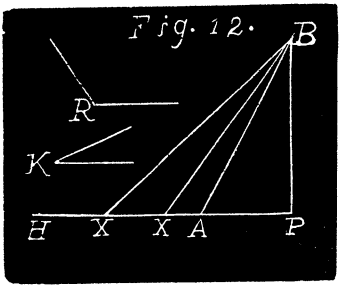
By GEORGE BRUCE HALSTED. A. M.. (Princeton); Ph. D., (Johns Hopkins), Member of the London Mathematical Society; and Professor of Mathematics in the University of Texas, Austin, Texas.

[Continued from the January Number.]

SCHOLION II. In the three preceding theorems I have studiously set down this condition, that the cutting straight AP, or XA, is understood to be of a *designated length as great as you choose.*

For if, without any determinate extent of the cutting straight it be discussed precisely concerning the exhibiting and demonstrating of the concurrence of two straights at the apex of a certain triangle, whose angles at the base are given (less indeed than two right angles) as, suppose, one right, and the other less than a right by as much as two degrees, or, if you please, by less: who is so devoid of geometry that he could not immediately show the thing itself demonstratively?

For suppose (fig. 12) given any angle BAP, as, say, 88 degrees. If therefore from any point B of this AB, is let fall on the base AP (Eu. I. 12.) the perpendicular BP, it holds certainly that in this triangle ABP would be exhibited demonstratively the desired concurrence in this point B.



But if the other angle at the base is postulated, and it less than a right, as suppose 84 degrees, which indeed the given angle K represents : then (Eu. I. 23.) one would be able to make toward the parts of the straight AB an equal angle APD, PD meeting this AB in D some intermediate point of it. Wherefore the desired concourse is again obtained demonstratively in this point D.

But finally: if the other angle is postulated obtuse, but yet less than 92 degrees, lest with the other given angle BAP it should make up two rights: : this may be represented in a certain angle R of 91 degrees. It is to be shown, that there is some one point X of this AP, to which the join BX makes an angle BXA equal to the given angle R of 91 degrees; so that therefore under a certain cutting straight AX the desired meeting in the point B many be obtained.

But we may proceed thus.

PA being produced to any point H, since the external angle BAH is (Eu. I. 13.) 92 degrees, since the interior angle BAP is by hypothesis 88 degrees; and again, (Eu. I. 16.) is greater not alone than the right angle BPA, but also, for the same reason, than any obtuse angle BXA, the point X being assumed wherever you choose within this PA, and indeed always growing

greater as the point X is assumed nearer to the point A, (Eu. I. 16.) : it is an evident consequence, that between those angles, one of 90 degrees at the point P, and the other of 92 degrees in the point A, one angle BXA is found, which is 91 degrees, truly equal to the given angle R. None the less, omitting this last observation about the obtuse angle, it is necessary most diligently to take care that the difficulty of this proposition [axiom] of Euclid be fixed in this, that it asserts the meeting of two straights; especially in that part in which they make with the cutting straight two angles less than two right angles; and assuredly that it asserts the aforesaid meeting thus, *of whatever length be the assigned transversal.*

For otherwise (as I have already mentioned in the preceding scholion) I will demonstrate that general meeting solely from the admitted meeting of this sort, when one of the angles is right; and indeed, even if it be admitted not for any assignable finite transversal, but alone admitted within the limits of any assigned very small transversal.

[To be continued.]

ARITHMETIC.

Conducted by B. F. FINKEL, Kidder, Mo. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

36. Proposed by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.

It costs $C=\$22$ to paper a room $a=18$ feet long, $b=15$ feet wide, and $c=10$ feet high, with paper $(m\diagup n)$th, $=\frac{3}{4}$, of a yard wide. Find the price of the paper per roll of $R=12$ linear yards.

Solution by the PROPOSER.

Making no allowance for "matching", the number of linear yards of paper required is $Y=\frac{1}{3}[2(a+b)c+ab](n\diagup m), =413\frac{1}{3}$; and, consequently, the number of rolls of paper required is $N=Y\diagup R=34\frac{4}{9}$.

Hence the price of the paper per roll is

$$P=\frac{3(m\diagup n)\mathrm{R}}{2(a+b)c+ab} \text{ of } \$C, =\$\frac{99}{175}=63\tfrac{7}{8} \text{ cents.}$$

Professor J. F. W. Scheffer, P. S. Berg, and J. A. Calderhead get $2.70 as the result. Professor John Faught and I. L. Beverage get $1.91⅓, and Professor T. W. Palmer gets $178⅓. These different results are due to different interpretations of the problem.